MATH 320 Unit 0 Exam Name:

Instructions:

Please write your answers on separate paper. Please write clearly and legibly, using a large font and plenty of white space (I need room to put my comments). Staple all your pages together, with your problems in order, when you turn in your exam. Make clear what work goes with which problem. Put your name on every page. To get credit, you must show adequate work to justify your answers. If unsure, show the work. No outside materials are permitted on this exam – no notes, papers, books, calculators, phones, smartwatches, or computers – only pens and pencils. You may freely use the contents of the box below, but not any other results we may have proved. Each problem is out of 10 points, 40 points maximum. You have 30 minutes.

1. Let $a, b, c, d \in \mathbb{Z}$ with $a|b$, $b|c$, and $c|d$. Prove that $a|d$.
2. Let $a, b, c, q, r \in \mathbb{Z}$ with $a = bq + r$. Prove that c is a common divisor of a, b if and only if c is a common divisor of b, r.
3. Let $n \in \mathbb{Z}$ with $n \geq 1$, and let $a, b_1, b_2, \ldots, b_n \in \mathbb{Z}$ with $a|b_1$, $a|b_2, \ldots, a|b_n$. Prove that $a|(b_1 + b_2 + \cdots + b_n)$.
4. Let $p, q \in \mathbb{Z}$ with p, q both prime. Prove that if $p|q$ then $|p| = |q|$.

Given $m, n \in \mathbb{Z}$, we say that m *divides* n, writing $m|n$, if there is some $k \in \mathbb{Z}$ with $mk = n$. We then call m a *divisor* of n.

Given $r, s, t \in \mathbb{Z}$, we say that r is a *common divisor* of s and t if r is a divisor of both, i.e. if $r|s$ and $r|t$.

Given $p \in \mathbb{Z}$ with $p \notin \{-1, 0, 1\}$, we say that p is *prime* if it satisfies:

$$\forall a, b \in \mathbb{Z}, \text{ if } p|ab \text{ then } \big(p|a \text{ or } p|b\big).$$

Bonus Theorem: Let $a, b \in \mathbb{Z}$ with $b \neq 0$. If $a|b$ then $|a| \leq |b|$.